Influence of Cell Geometry on Division-Plane Positioning

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SUMMARY

The spatial organization of cells depends on their ability to sense their own shape and size. Here, we investigate how cell shape affects the positioning of the nucleus, spindle and subsequent cell division plane. To manipulate geometrical parameters in a systematic manner, we place individual sea urchin eggs into microfabricated chambers of defined geometry (e.g., triangles, rectangles, and ellipses). In each shape, the nucleus is positioned at the center of mass and is stretched by microtubules along an axis maintained through mitosis and predictive of the future division plane. We develop a simple computational model that posits that microtubules sense cell geometry by probing cellular space and orient the nucleus by exerting pulling forces that scale to microtubule length. This model quantitatively predicts division-axis orientation probability for a wide variety of cell shapes, even in multicellular contexts, and estimates scaling exponents for length-dependent microtubule forces.

INTRODUCTION

The orientation of the division plane is a key element in the generation of a multicellular organism. During development, cells adopt a wide variety of geometrical configurations including spherical, ellipsoidal, and polyhedral shapes. Cell shape is thought to dictate the orientation of the division plane in many systems (Concha and Adams, 1998; Gray et al., 2004; O’Connell and Wang, 2000; Strauss et al., 2006; Thery and Bornens, 2008). This effect may guide the polarity of the initial cleavages in many developing embryos (Jenkinson, 1909). The correlation of the division plane with cell shape is described in Hertwig’s empirical rule, also referred to as the “long axis rule” (Hertwig, 1884): “The two poles of the division figure come to lie in the direction of the greatest protoplasmic mass.” The mechanism of nuclear and spindle positioning is now known to be a dynamic process that involves motor proteins, pulling and/or pushing forces from the microtubule (MT) and/or actin cytoskeletons (Grill and Hyman, 2005; Kunda and Baum, 2009; Reinsch and Gonczy, 1998; Wuhr et al., 2009). Depending on cell type, the division plane can be set by the orientation of the nucleus during interphase or early prophase or may be modified by rotation or movement of the spindle during anaphase. How these force-generating systems globally sense the shape and dimensions of the cell remains an outstanding question.

The single-cell sea urchin zygote is an attractive cell type for studying the effects of cell geometry. To date, many well-characterized systems for studying spindle positioning are in cells that exhibit asymmetric division, such as in C. elegans or S. cerevisiae, or in adherent mammalian cells. In these cell types, polarity cues or cell adhesion patterns appear to override geometric cues (Carminati and Stearns, 1997; Grill et al., 2001; Thery et al., 2005). In contrast, sea urchin zygotes are nonadherent, divide symmetrically, and appear to lack extrinsic polarity cues. These are spherical cells that have a highly reproducible cell size and cell-cycle timing. They have been used extensively in seminal studies in cytokinesis using physical manipulation approaches (Rappaport, 1996).

Here, we introduce the use of microfabricated wells to manipulate cell geometry parameters in a systematic and quantitative manner. By placing the sea urchin eggs into these chambers, we can mold them into highly reproducible series of cell shapes. In contrast to traditional physical manipulation methods on single cells, this approach allows for rapid acquisition of large datasets suitable for quantitative analysis. We find that the “long axis rule” does not apply for certain cell shapes. We develop a computational model that fully predicts the preferred division plane and the probability that this axis will be chosen for any given cell shape. This work demonstrates that cell shape sensing can be explained by a simple mechanism based upon microtubule length-dependent forces.

RESULTS

Manipulation of Cell Shape using Microfabricated Chambers

To control the geometry of sea urchin zygotes, we devised polydimethylsiloxane (PDMS) microfabricated chambers in which single eggs could be pushed into a variety of defined shapes.
The total volume of each chamber was kept similar to the egg volume whereas the height was smaller than the egg diameter, so that the egg was slightly flattened into its new shape, allowing a bidimensional description of the process (see below). Cells were malleable and could form relatively sharp angles (down to 5–10 μm local radii of curvature), although further deformation was limited, probably because of cortical tension (Figure 1B). By removing sea water between the PDMS array and the top coverslip, we could cause the eggs to enter the chamber and change their shape in 1–5 min (Figure 2A). The embryos were surrounded by sea water and were not deprived of oxygen, as PDMS is gas permeable. In all shapes assessed, cells went on to divide within the chambers with normal timing for at least 24 hr (Figure S1B), indicating that their general physiology was not grossly perturbed.

Cell Shape Dictates Division-Plane Positioning during Interphase

We monitored the positioning and orientation of the division plane in many different shapes. In some shapes, cells divided in a plane at the cytoplasmic center perpendicular to the longest axis of the cell at this center. However, in other cases, we found exceptions to the long axis rule. Cells often divided at an angle different from that perpendicular to the cell’s longest axis, for instance in an ellipse with a small aspect ratio in which the long axis was not well defined. The rule also did not apply well to shapes such as squares or rectangles. Rectangular cells usually did not divide along the long axis, which is the diagonal in this shape, but rather along the longest axis of symmetry (Figure 1C). To gain more insight into this process, we performed time-lapse imaging of embryos inside chambers going through the first cell cycle (from typically 20 min after fertilization to the end of cytokinesis). DNA was stained with Hoechst. Note the early centering of the zygote nucleus after pronuclei migration and fusion, and the elongation of the interphase nucleus along the future division axis. The orientation of the interphase nucleus predicts the future spindle axis and division plane in these cells. The relative centering and orientation at metaphase (M), anaphase (A), and cytokinesis (C) relative to interphase (I) are computed as indicated in the figure from time-lapse sequences. Error bars represent standard deviations. Scale bars, 20 μm. See also Figure S1, Figure S2, and Movie S1.
formed in a plane perpendicular to this axis (Figures 1D and 1E, Figure S2A, and Movie S1). Importantly, the furrow did not appear to reorient or reposition during contraction (Figure S2B). After cytokinesis, however, adhesion forces between the two daughter cells sometimes led to a minor (usually < 5–10°C14) reorientation of the initial division axis. Computer-aided measurements showed that the position of the interphase nucleus predicted the position of the mitotic spindle and the subsequent division plane (to within 5% of the cell’s radius on average), and that orientation of the interphase nucleus also predicted the orientation of the spindle and division plane (within 10% on average) (Figure 1E). Together these data suggested that the division position and axis relative to a given geometry are set by the orientation of the nucleus during interphase or early prophase.

**Nuclear Centering Is Dependent on Microtubules and Not Actin**

The microtubule and actin cytoskeletons have been implicated in positioning and orienting the nucleus and spindle in various cell types and contexts (Carminati and Stearns, 1997; Grill et al., 2001; Schuh and Ellenberg, 2008; Tran et al., 2001). In normal urchin zygotes, interphase microtubules (MTs) are organized in two asters nucleated from two diametrically opposed zones around the nucleus (Foe and von Dassow, 2008; Holy and Schatten, 1997). F-actin appears relatively diffuse throughout the cytoplasm and is enhanced at the cell surface (Wong et al., 1997). Inhibition of MTs with nocodazole or F-actin with Latrunculin B blocks pronuclei migration or entry into mitosis, depending on the time of addition (data not shown) (Hamaguchi and Hiramoto, 1986; Schatten et al., 1986). To investigate the mechanism of nuclear centering, after pronuclear fusion, we pushed cells into chambers to dynamically alter their cell shape and then assayed for the ability of the nucleus to center relative to this new shape (Figure 2A). In control cells, the nucleus recentered at the new center of mass in less than 1–2 min (Figures 2A–2C). Addition of 20 μM nocodazole prior to the cell shape change inhibited recentering of the nucleus (Figures 2B and 2C); this nocodazole treatment led to the depolymerization of detectable MTs at this stage (Figure 2D). Depolymerization of F-actin with 20 μM Latrunculin B did not affect the process (Figure 2C and Figure S3C). Thus, nuclear centering depends on microtubules and not actin in this cell type.

**The Nucleus Acts as a Force Sensor**

We observed that nuclear shape was elongated along an axis that predicts the future spindle axis. The nucleus elongated along the new long axis in less than 3 min after the change in cell shape (Figure 2A). Nocodazole treatment prior to the shape change abolished nuclear elongation and gave rise to a spherical nucleus (Figures 2B and 2D and Figure 3A). Treatment with 20 μM nocodazole B had a minor but significant effect on the nuclear aspect ratio (Figure 2B and Figure 3A). Depolymerization of both MTs and actin was similar to treatment with nocodazole alone, suggesting that these effects were not additive. Latrunculin B did not grossly affect MT distribution (Figure S3D). Based upon work in other cell types, F-actin could contribute to multiple aspects of nuclear elongation, including
the linking of the centrosomes to the nuclear envelope, mechanical properties of the nucleus, and the ability of MT motors to pull (Bettinger et al., 2004; Kunda and Baum, 2009). In these urchin cells, MTs provide the primary force that elongates the nucleus, with actin providing a minor contribution.

We found that nuclear elongation was more pronounced in cells with elongated cell shapes (Figure 3B). We examined this correlation in a series of ellipsoidal and rectangular cells with increasing aspects ratios. Quantitation of the nuclear and cell aspect ratios showed a significant positive correlation between these parameters in both of these classes of shapes ($R^2 = 0.88$ and 0.98 for ellipses and rectangles, respectively).

We considered whether the degree of nuclear elongation could correspond to a stretching force exerted on the nucleus. Previous biophysical measurements have shown that the nucleus in *Xenopus* behaves as an elastic material with a defined elastic surface modulus of 25 pN/μm (Dahl et al., 2004). Nocodazole treatment caused the elongated nuclear shape to become spherical within 5 min (Figures 3D–3G). This effect was independent of initial nuclear aspect ratio. This behavior illustrates the elastic nature of the nucleus and the absence of plasticity (Dahl et al., 2004, 2008; Vaziri and Mofrad, 2007). These results indicate that MTs exert forces to stretch the elastic nucleus along the long axis of the cell, and that these forces scale with the aspect ratio of the cell shape.

Estimates of MT forces on the nuclear envelope can be obtained by representing the nucleus as a spherical thin elastic shell of elastic surface modulus $K$ and radius $r_N$. The force necessary to deform such sphere into a prolate spheroid of aspect ratio

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Figure 3. Nuclear Shape Is an Indicator of Microtubule Pulling Forces

(A) Quantification of nuclear shape in cells treated with the indicated drugs (as in Figure 2B). The nuclear aspect ratio is defined as the ratio between the long and short axis of the ellipsoid shape of the nucleus. The cells used for this quantification have a geometrical aspect ratio smaller than 1.5 (see 3C). Error bars represent standard deviations.

(B) Nuclear shape in cells with increasing aspect ratios. Close-up images of nuclei in different cells are presented on the right. Dotted colored lines outline each nucleus. Their superimposition highlights the increase in nuclear aspect ratio as cells are more elongated. Black arrows in the bright-field picture point at aster centers on the side of the nucleus.

(C) Plot of the nuclear aspect ratio as a function of the cell aspect ratio, $a/b$, for a series of ellipsoidal and rectangular cell shapes. Each point is binned from data on 10 or more cells having the same shape, for a total number of 104 cells for the ellipses and 82 for the rectangles. Error bars represent standard deviations. The dotted lines are depicted to guide the eyes.

(D) Time-lapse images of the interphase nucleus in a cell just prior to and after treatment with 20 μM nocodazole. Note that nuclear shape becomes spherical in minutes.

(E) The nuclear aspect ratio as a function of time in the representative cell in Figure 2D and two other cells treated in the same manner.

(F) Effect of 20 μM nocodazole on the nuclear shape in elongated cells inside chambers. Black arrows point at aster centers on the side of the nucleus and the dotted purple lines outline the nucleus.

(G) Nuclear aspect ratio in cells inside chambers before and after treatment with 20 μM nocodazole (1 cell per color, $n = 8$ cells). The black horizontal bar represents the mean value.

**p < 0.01, Student’s t test compared with the control. Scale bars, 20 μm.**
Figure 4. A Computational Model that Predicts Nuclear Orientation and Division-Plane Orientation in Response to Cell Geometry

(A) Schematic 2D representation of the cellular organization used for modeling. The nucleus (gray) is located at the center of mass and oriented along an axis \( a \). Microtubules (green) emanate from two centrosomes (orange) attached to each side of the nucleus and extend out to the cortex. The total force generated by the two MT asters along the \( a \) axis is \( F(a) \), and the total torque at the cell’s center is \( T(a) \). Inset: A microtubule in the aster has a length \( L \) and is nucleated at an angle \( \theta \) from the axis \( a \). It produces a pulling force \( f \) at its nuclear attachment and a torque \( t \) at the nucleus center. The projection of the force along the axis \( a \) is denoted \( f_p \).

(B) Examples of cells in specific geometries (triangles and fan) at interphase and after cytokinesis, stained with Hoechst. The nuclear and subsequent spindle orientation, \( a \), is reported and highlighted by the yellow dotted line.

(C) The plots represent the different outputs of the model for these three cells in (B) (corresponding colors). The dots on the plots position the experimental spindle orientation \( a_{\text{exp}} \). Note that in the three cases, \( a_{\text{exp}} \) is close to maxima of the total normalized force and the probability density, to zeros of the total normalized torque, and to minima of the normalized potential.

(D) (Left) Frequency histogram of the absolute difference between \( a_{\text{exp}} \) and \( a_{\text{th}} \) (calculated as the maxima of the probability density) for 77 dividing cells with different shapes. (Right) Frequency histogram of the probability density ratio for the same 77 sequences. The ratio is 1 when the experimental axis has the same probability density as the theoretical axis and 0 when the experimental axis falls in the zone where the probability density is 0.

(E) Model prediction of cleavage-plane orientation probability density in the depicted shapes and experimental frequency histograms of division axis in the depicted shape. Note that the division-plane angle used here, \( a_{\text{div}} \), is rotated 90° from the nuclear orientation angle \( a \) presented in panels A–D.
$p_N$ is given by (see supplemental model, Extended Experimental Procedures):
\[ F = \frac{K}{1 + \nu} \ln \frac{r_N}{r_C} \left( \rho_N^{2/3} - 1 \right), \]  
(1)

where $\nu$ is the Poisson ratio of the material and $r_C$ is the radius of the force application zone. If we assume that the elastic surface modulus of the sea urchin nucleus is similar to that of the Xenopus nucleus, our calculations estimate that the MT-dependent forces on the nucleus range from 10 to 30 pN, depending on the shape of the cell.

**Microtubule Organization in Different Cell Shapes**

We next examined the distribution of the interphase microtubules responsible for nuclear positioning. Bright-field images show that during interphase, the nucleus is associated with two asters, one on each side of the nucleus (Figure 2B and Figures 3B and 3F). To examine MTs directly, we performed immunostaining of tubulin in cells in the chambers (Figures S3A and S3B; see Experimental Procedures). In all shapes observed, MT staining confirmed the bipolar aster organization around the nucleus (Figure 2D and Figure S3E). MTs filled the whole volume of the cell and extended out to the cortex, even in elongated cells. They appeared to emanate from the centrosome at a relatively constant angular density, and the complete aster extended out over a little more than 180° (Figure S3F). In general, MTs did not exhibit buckling or curling around the cortex. In some cases there appeared to be increased tubulin staining, often near the cortex and regions of cell elongation. This pattern may be an artifact of fixation, in which some regions of the cells may be fixed unevenly due to staining in the chambers (Figure 2D and Figure S3E); alternatively, it may represent true increased local density of MTs in these regions. Although MT probes for imaging MT organization in live cells have been recently described, that study used injection of esconsin mRNA to assay 8-cell-stage embryos (von Dassow et al., 2009); our preliminary attempts to inject live 1-cell embryos in chambers with labeled esconsin protein, without modifying their shape, have not been successful (data not shown). The distribution of MTs throughout the cytoplasm and cortex suggest that MTs may directly probe the whole cell surface or volume for shape sensing.

The stretching of the nucleus and the lack of MT buckling strongly suggest that MTs are providing pulling forces on the nucleus (Figure 2D and Figure S3E). Although MT pushing, which results from MT polymerization, is a major factor in smaller cells such as fission yeast (Dogterom and Yurke, 1997; Tran et al., 2001), it may be only a minor component in these larger cells (Wuhr et al., 2009).

**Computational Models for Microtubule-Based Shape Sensing**

To understand how microtubules may globally sense cell shape for nuclear positioning, we developed quantitative models. The model aims to compute the forces and torques generated by MTs on the nucleus during interphase and output the probability of a given orientation of the division axis in a given geometry (Thery et al., 2007). We use a two-dimensional representation in which MTs emanate from two points diametrically opposed around the nucleus, grow straight, and reach out the cortex. Each microtubule produces a pulling force, $f$, on its nuclear attachment site and a torque, $\tau$, at the nucleus center (Figure 4A).

A necessary input of the model is to assume that the force generated by each microtubule, $f$, depends on its length $L$ (the exact nature of this dependence is discussed and tested below). Through this assumption, it can be shown that the nucleus will be centered near the cytoplasmic center of mass (Bjerknes, 1986; Howard, 2006). Thus, in what follows, the nucleus is assumed to be centered at the center of mass of the geometry, and we focus our attention on describing how cell shape affects axis orientation.

For each shape, we aim to compute the global force $F$ and torque $T$ generated as a function of the orientation angle of the stretched nuclear axis, $\alpha$ (Figure 4A). For each possible orientation $\alpha$ (varying from 0 to $\pi$), we generate two asters of $N$ MTs nucleated at a constant angular density $\rho$ from centrosomes placed at a distance $r_C$ from the cell’s center of mass along the axis. An MT oriented along the direction $\alpha + 0$ has a length $L(\alpha, \theta)$ and generates a pulling force $f(L(\alpha, \theta))$ on its nuclear attachment that we project on the axis $\alpha$, to compute the noncompensated force $f_p$:

$$f_{p}(\alpha, \theta) = f(L(\alpha, \theta))\cos(\theta).$$  
(2)

The resultant total force $F(\alpha)$ generated by each aster on its nuclear attachment along the axis $\alpha$ is then obtained by summing the projected force over all MTs:

$$F(\alpha) = \int_{0}^{\pi} f(L(\alpha, \theta))\cos(\theta) d\theta.$$  
(3)

where $\Phi$ is the total angular width of the aster. The torque created by each MT at the center of mass $O$ and projected along the $z$ axis is in turn computed as:

$$\tau(\alpha, \theta) = r_C f(L(\alpha, \theta))\sin(\theta),$$  
(4)

which yields a total torque, $T(\alpha)$,

$$T(\alpha) = r_C \int_{0}^{\pi} f(L(\alpha, \theta))\sin(\theta) d\theta.$$  
(5)

Initial tests of the model showed that, above a certain threshold, the number of MTs $N$ (or equivalently the angular density: $\rho = N/\Phi$) does not impact axis definition but only affects the time required to align along this axis (see below). Thus, in what follows, we keep $N$ as a silent parameter by normalizing...
the total force and torque with the total force computed in a round normal cell, $F_0$: $\bar{F}(\alpha) = F(\alpha)/F_0$ and $\bar{T}(\alpha) = T(\alpha)/(n_RF_0)$. These normalized parameters have no units and are independent of MT number. Stable axis orientation can be identified from local minima of the normalized potential $V(\alpha)$ computed as a primitive of $\bar{T}(\alpha)$. The probability density associated with each orientation $p(\alpha)$ is then calculated by introducing a white noise in the distribution of torques as proposed in Thery et al. (2007),

$$p(\alpha) = p_0 \exp \left( -\frac{V(\alpha)}{C} \right). \quad (7)$$

In this formula, $p_0$ is adjusted so that $\int p(\alpha) = 1$ and $C$ is a dimensionless fitting parameter that includes noise strength and nuclear friction (Thery et al., 2007). The parameter $C$ is fitted by dichotomy to match the experimental average orientation in a ellipse with a small aspect ratio (see Figure 4F). This $C$ value, $eta_{\text{max}}$, is the geometry of the cell (see supplemental model, Extended Experimental Procedures and Table S1).

As examples, we present the results of two triangles and an elongated fan-like shape, for which we plot $\bar{F}(\alpha)$, $\bar{T}(\alpha)$, and $p(\alpha)$ (Figures 4B and 4C). In this cases, the theoretical orientation angle $\alpha_{\text{th}}$, which is calculated at the highest predicted orientation probability, was within 5%–10% of the experimental orientation angle $\alpha_{\text{exp}}$ measured at anaphase.

**Testing the Model for Different Shapes**

We compared theoretical predictions and experimental results of time-lapse sequences of cells in many different shapes. In general, the theoretical predictions closely approximated the experimental results. In a large dataset of 77 cells of assorted shapes, the average difference between the experimental and theoretical angle $\alpha_{\text{exp}}$ and $\alpha_{\text{th}}$ was $15.6 \pm 10.0$ (Figure 4D). In addition, we also tested whether the model predicts the probability that a specific orientation will be chosen. In shapes where length differences between different axes are small (a circle), a large difference between theoretical and experimental angles could be obtained whereas the difference in probability density is close to 0, as all orientations have an equal probability. The average ratio between the theoretical and experimental probability densities obtained from the same set of 77 cells was $0.72 \pm 0.16$ (Figure 4D).

We also compared sets of division-plane data for each individual cell shape (Figure 4E). In each shape, the computed probability density quantitatively correlated with the experimental division-plane orientation frequency. Thus, the model was highly successful in predicting the division-plane orientations.

**Testing the Sensitivity of Shape Sensing**

We sought to determine how sensitive a cell is in sensing its shape. We generated series of datasets in which the aspect ratios of the shape were systematically altered (Figure 4F). The simplest case is a series covering the transition from a circle to ellipses with increasing aspect ratio. As expected, the division axis was purely random in a circular cell. Shaping a cell into an ellipse with an aspect ratio as small as 1.15, however, was sufficient to significantly bias the division orientation perpendicular to the long axis. At higher aspect ratios, the average orientation saturated, and the standard deviation decreased. Similar results were seen in the transition from a square to rectangles. Another informative case consisted of varying the ratio between the two major axes of a fan-like shape, in which the transition between two orthogonal division axes can be examined. This transition showed a sharp slope at the point where the two axes have similar length, supporting the view that the shape-sensing mechanism is finely tuned. These results demonstrate that the cell can robustly sense fine differences in aspect ratio of less than 15%.

In all cases, the results of the theoretical model using a single set of fixed parameters matched the experimental data with remarkable accuracy in predicting the average orientation, the standard deviation, and the precision in sensing shape. Thus this simple model, which improves upon Hertwig’s long axis rule, demonstrates that length-dependent MT forces can account for spindle orientation and cell shape sensing.

**Testing Mechanisms of Microtubule-Dependent Forces**

MT pulling forces can be generated through minus-end-directed motors such as cytoplasmic dynein that may interact with the cortex and/or with cytoplasmic elements that serve as a putative “cytoplasmic scaffold” (Carminati and Stearns, 1997; Gonczy et al., 1999; Hamaguchi and Hiramoto, 1986; Kimura and Onami, 2005; O’Connell and Wang, 2000; Juhász et al., 2010). We considered various models that differ in the location and density of MT pulling motors. One distinguishing hallmark between these models is how the force scales with MT length $L$ (Hays et al., 1982). For instance, in a model in which motors are attached to the cortex and are in limiting concentration, the MT force is predicted to scale approximately with $L^2$ (Grill and Hyman, 2005; Haraj and Hyman, 2009; Howard, 2006). In a model in which motors are uniformly attached to the cytoplasmic matrix, the pulling force may scale with $L$. Thus, to test mechanisms of length-dependent forces, we used a general scaling law with the form of $f(L) \sim L^\beta$ and sought to determine an experimental value of the exponent $\beta$.

This $\beta$ value could be estimated by comparing our experimental data on nuclear stretching with theoretical pulling forces exerted by MTs. We focused on the data series of ellipsoid and rectangular shapes with increasing aspect ratio (Figure 3B). In both of these types of shapes, the preferred axis orientation corresponds to $\alpha = 0$ and to the maximum total force as well, $F_{\text{max}} = F(\alpha = 0)$ (Figure 5). We thus computed for different values of $\beta$ the evolution of $F_{\text{max}}$ as a function of the aspect ratio $a/b$, in ellipses and rectangles (see different color plots in Figure 5), and calculated what value of $\beta$ fits best the experimental behavior (see supplemental model, Extended Experimental Procedures). This approach leads to exponent values for each dataset: $\beta = 3.1 \pm 0.9$ and $\beta_{\text{max}} = 3.4 \pm 1.5$. Integrating MT forces on a 3D volume led to estimates of exponents of 4 to 4.5 (see supplemental model, Extended Experimental Procedures). Thus, these two independent datasets show that MT forces roughly scale with $L$ to the power of 3 to 5. This measurement indicates nonlinearity in these MT length-scaling forces. This analysis rules out...
certain mechanisms, such as a simple model where cytoplasmic-anchored motors saturate the MT lattice, and suggests mechanisms that incorporate more complex length dependency (see Discussion).

We also tested whether these different power laws could affect orientation axis, by comparing experimental division-axis distribution with theoretical density probability plots generated with different $\beta$ values (Figure S4). This comparison showed that axis orientation was independent of $\beta$ (for $\beta > 0$) and suggested that neither the total force intensity nor the detailed scaling of the force-generation system are critical for steady-state orientation (see also Figure 7). We additionally tested another proposed shape-sensing mechanism based on contact-angle-dependent MT forces (Tsou et al., 2003) and found that it was not consistent with our experimental results in some shapes (Figure S4).

Predicting Division Axis in Multicellular Contexts

To test the generality of this model, we examined whether it could predict division planes in multicellular, developmental contexts. First, we used it to predict division positions in subsequent embryonic divisions in the sea urchin. We followed the behavior of urchin embryos through multiple divisions as they were confined in PDMS chambers of different geometries (Figure 6A). Cells were shaped by contact with the wall of the chamber and by other cells in the chamber. We used our model to predict the division planes based solely upon cell shapes and found that the theoretical predictions were in excellent agreement with the experimental findings: the average difference found that the theoretical predictions were in excellent agreement with the experimental findings: the average difference between the experimental and theoretical division-axis orientation was $11.5^\circ \pm 9.1^\circ$ and the mean ratio of theoretical to experimental probability densities was $0.82 \pm 0.12$.

We note that these experiments mimic geometrical effects during embryonic development in different organisms, in which the chamber can be regarded as an artificial egg shell that constrains and shapes the dividing cells. A circular chamber corresponds, for instance, to the situation for most deuterostomes including humans and urchins, as well as the animal pole view of meroblastic cleavages of Xenopus or Zebrafish eggs. An ellipsoidal chamber is similar to a mouse embryo or an embryo of the nematode Prionchulus, which exhibits initial symmetric cleavages (Schierenberg, 2006). The division planes with sea urchin blastomeres in the chambers reproduced with excellent fidelity the observed patterns of initial blastula divisions seen in these diverse organisms. Although cell-cell contact has been considered to provide spindle orientation cues (Goldstein, 1995; Wang et al., 1997), we noted that in certain shaped chambers (like triangles and elongated rectangles), spindle orientation did not correlate with the orientation of the first cleavage plane and thus appeared to be governed in these situations more by cell shape cues than by putative cell-cell contact signals.

Next, we asked whether our model could predict spindle axis orientation within an adult tissue. Figure 6B depicts an illustration of a tissue section from the pigeon testis (Guyer, 1900), which shows the shape and spindle orientation of spermatocytes undergoing meiosis. From this image, we traced the geometry of each cell and simulated the probability density distribution of spindle orientations. Our model was able to predict the division planes accurately: the average difference between the experimental and theoretical spindle orientation angle was $8.5^\circ \pm 3.1^\circ$, and the mean ratio of theoretical to experimental probability densities was $0.79 \pm 0.22$ (n = 13). Thus, in these cells, spindle orientation may be determined by geometrical cues. These findings demonstrate how this model may be generally applicable in predicting division patterns and illustrate how cell shape can be a major parameter in defining cleavage patterns during development.

Shape Sensing in Spindle Reorientation

In many cell types, the spindle undergoes rotation during metaphase or anaphase. Although the spindle does not normally rotate, in these early divisions in sea urchins (Figure 1), we tested...
whether it could reorient upon a change in cell shape. We arrested urchin embryos in metaphase by treatment with the proteasome inhibitor MG132 and introduced them into chambers of different shapes. We observed that the metaphase spindle reoriented along an axis defined by the new cell shape (Figures 7A–7E and Figure S6). The relatively slow rotation allowed us to analyze this movement using time-lapse imaging. Rotations occurred in a relatively unidirectional and steady movement and occurred in 3 to 30 min depending on the initial axis orientation and the geometry. Spindles that were properly aligned initially did not exhibit rotations or oscillations. This process depended on microtubules but not on actin (Figures 7C, 7D, and 7E). The slow timescale for this rotation as compared to the interphase nucleus, which orients in less than 3 min (Figure 2A), suggested that the torque intensity generated by MTs was much smaller in metaphase (see supplemental model, Extended Experimental Procedures). We studied this movement using our computational model (Figure 7F), using an identical set of parameters (except for MT number, see below) and an estimate of friction of the nucleus and spindle in the cytoplasm (see supplemental model, Extended Experimental Procedures). The results of the simulations correlated well with experimental findings (Figure 7B and Figure S6). The best overall fit in the model occurred if we assumed that metaphase spindles have only around 0.6% of the number of effective MTs of the interphase arrays (at which the ratio of interphase to metaphase number of MTs is around 150; Figure 7G). The calculation is consistent with immunofluorescence images showing very few astral MTs contacting the cortex in these cells (Figures 7C and 7D) (Strickland et al., 2005). Thus, this model is applicable to spindle orientation as well as nuclear orientation.

Further, this example demonstrates how this model can be used to analyze parameters such as the rate of orientation and the number of effective MTs.

**DISCUSSION**

**A Model for Orienting the Division Axis Relative to Cell Shape**

By systematically studying the effects of changing cell shape, we develop and test a simple quantitative model for how cell geometry dictates the positioning and orientation of the nucleus and spindle, which subsequently positions the cleavage furrow. MTs emanating from centrosomes on the nucleus probe the dimensions of the cell and exert pulling forces that depend on MT length. The observed elongation of the nuclear envelope provides estimated forces on the order of 10–30 pN on the nuclear envelope, which scales with the aspect ratio of the cell. This force, which may represent a time average of a more dynamic molecular organization, corresponds to a relatively small number of force generators (around 10–50), as seen in other systems (Grill et al., 2003). Through modeling, we demonstrate that such length-dependent MT forces are sufficient to explain nuclear centering and orientation in these cells; more elaborate mechanisms involving intracellular gradients (Moseley and Nurse, 2010) or actin-based mechanisms (Kunda and Baum, 2009) are not needed. The cell may thus sense its shape primarily by "measuring" the length of its MTs that extend from the centrosome to the cell surface. Integration of these forces over the whole cell provides a mechanical ensemble that seeks to reach equilibrium.
Our theoretical model is kept as simple as possible and deliberately infers only fixed adjustable parameters, modeling all cells in different geometries in the exact same manner. The exceptional fit between our experimental data and theoretical model for a wide variety of different shapes indicates that this process is close to the theoretical limit. Our model can readily predict spindle orientation with good accuracy in other cell types and thus promises to be adaptable for predicting division orientations in many cell types in a broad variety of contexts. Cell-type-specific parameters such as the relative sizes of the cell and nuclei, as well as MT organization, could even be optimized to provide the best predictability (Figure S5). These rules may apply best to cells in which cell geometry itself plays a primary rule in determining the axis of division. In addition, this model could be valuable to predict whether a given cell uses cell geometry as a primary cue and to analyze parameters in the system, such as effective MT number.

**Microtubule Length-Dependent Forces**
A key aspect of our model is the assumption that MTs exert forces that scale with MT length. Recent progress has begun to reveal molecular details of how such length-dependent forces could occur through interactions with MT motors such as dynein or depolymerizing kinesin (Gardner et al., 2008; Tischer et al., 2008).

*Figure 7. Rotation of the Metaphase Spindle in Response to Cell-Shape Changes*
(A) Cells were blocked in metaphase by treatment with the inhibitor MG132 and then changed into a new shape by introducing them into a well. Time-lapse images of spindle rotation (as shown by Hoechst staining) in a metaphase-arrested cell in a rectangular chamber are shown.
(B) Examples of experimental and theoretical plots of the reorientation of the spindle axis to the theoretical force axis. The theoretical plots are computed by assuming a ratio of metaphase to interphase MT number of 1/150 (see Figure 5G).
(C) Images of embryos blocked in metaphase and introduced into a chamber for 20 min, in the presence and in the absence of 20 μM nocodazole. The red point corresponds to the cell’s center of mass and the yellow dotted line corresponds to the spindle axis.
(D) Confocal images of embryos blocked in metaphase, treated with 1% DMSO or 20 μM nocodazole and introduced into a chamber. After 20 min, cells were fixed and stained in situ for tubulin (green) and DNA (blue). Images are projections of Z stacks of 20 mid-section slices that cover a total depth of 10 μm.
(E) Quantification of spindle centering and orientation along the theoretical force axis in response to shape changes, in the indicated conditions. The error in centering is defined as the ratio between the distances from the metaphase spindle center to the cell’s center of mass to the long axis of the cell. The orientation ratio, \( p(\theta_{\text{exp}})/p(\theta_{\text{th}}) \), is 1 when the spindle axis has the same probability density as the theoretical axis and 0 when the spindle axis falls in the zone where the probability density is 0. Error bars represent standard deviations.
(F) Schematic 2D representation of the cellular organization at metaphase.
(G) Average error between experimental and theoretical times corresponding to 12 different cells, plotted as a function of the ratio of metaphase to interphase MT number. A positive number corresponds to an underestimation in the reorientation timing of the model whereas a negative number corresponds to an overestimation of the model.

*“p < 0.01, Student’s t test compared with the control. Scale bars, 20 μm. See also Figure S6.*
The L3 motors, which are in limited numbers in the cytoplasm, sensing model that is more consistent with our scaling close to 424 Cell (2009b), and Velve-Casquillas et al. (2010). A SU-8-positive master containing Chambers containing the arrays of microwells were fabricated by rapid proto-

Microchamber Fabrication and Operation

EXPERIMENTAL PROCEDURES

Motors may be situated in the cytoplasm, for instance on some internal membrane component or some putative cytoplasmic “matrix.” Alternatively, motors may primarily associate on the MT lattice and travel to accumulate at the plus ends.

Although technical limitations in the sea urchin system have limited the ability to directly image motors in this process, our experimental data coupled with computational modeling allow us to discriminate between some proposed mechanisms. Our results estimate that the forces scale in a nonlinear manner to MT length (L) to roughly $L^{3}$. This finding is not consistent with basic linear models in which the number of force-generating elements is strictly proportional to MT length. One previously proposed nonlinear scaling can result from having a limited number of motors at the cortex, which pull on a fraction of astral MTs (Grill and Hyman, 2005; Howard, 2006). This view makes the aster “surface sensitive” and is equivalent to a force per MT scaling with $L^{2}$ (Hara and Kimura, 2009). We propose a “volume sensing” model that is more consistent with our scaling close to the $L^{3}$: motors, which are in limited numbers in the cytoplasm, encounter a given MT of length L with a probability that is proportional to the cone-shaped unit volume surrounding the MT.

Additional dynamic sources of nonlinear length scaling may be positive feedback or cooperativity among multiple mechanisms. For instance, longer MTs might accumulate more motors not only because of their increased length but also because of their long lifetime (Seetapun and Odde, 2010), which can lead to increased tubulin posttranslational modifications that can help recruit more motors and increase accumulation of MT stability factors (Cai et al., 2009). Another potential factor may be the increased probability of new nucleation of noncentrosomal MTs off of longer pre-existing MTs (Janson et al., 2007). Further quantitative analysis of MTs and associated motors will help to test these proposed models.

EXPERIMENTAL PROCEDURES

Microchamber Fabrication and Operation

Chambers containing the arrays of microwells were fabricated by rapid prototyping and PDMS technology as described in Minc et al. (2009a), Minc et al. (2009b), and Velve-Casquillas et al. (2010). A SU-8-positive master containing hundreds of posts that are 70 μm in height and of different geometries was first made by micro lithography (Figure S1A). A 10:1 mixture of PDMS Sylgard 184 silicone elastomer and curing agent was poured onto the master and baked at 65°C for 4 hr. The replica was cut, peeled off the master, and activated with a plasma cleaner (Harrick Plasma). A 100 μl drop of fertilized eggs in sea water was placed onto the PDMS replica and the eggs were left to sediment by gravity for 2 min. A 22 mm² glass coverslip was then placed on top of the suspension, and water was gently sucked from the sides of the coverslip with a kimwipe, which slowly pushed the eggs into the chambers. To perfuse cells inside chambers, for in situ immunofluorescence or drug treatment, we used an inverse set-up: The PDMS replica was first pierced with two large holes. The drop of eggs was placed onto a 45 × 50 mm³ large coverslip and subsequently covered with the PDMS replica. Water was removed, as before, to reduce the space between the PDMS and the coverslip, which caused the eggs to adopt the shape of the microchamber (Figure S3A). Buffers, drugs, or fixatives were added slowly through the large holes and removed by sucking from the side of the PDMS replica. Cells were monitored on the microscope to ensure that no major shape change was occurring. For incubation periods longer than 1 hr the whole chamber was placed in a sealed box filled with some wet paper to limit drying. Rapid nocodazole treatment on normal spherical eggs was performed in a different set of microfluidic channels designed as in Minc and Chang (2010).

Collection and Fertilization of Sea Urchin Eggs

Lytechinus pictus sea urchins were purchased from Marinus Scientific. Gametes were collected by intraocoelemic injection of 0.5M KCl. The eggs were resuspended and gently agitated twice in fresh sea water. Sperm were diluted 1000-fold in sea water, activated by vigorous aeration, and then added dropwise to the eggs. Fertilization was monitored after 2 min. Fertilization envelopes were subsequently removed by pouring the eggs through Nitex mesh in sea water with 5 mM PABA (4-Aminobenzoic acid). The cells were maintained at 17°C–19°C throughout the experiment.

Pharmacological Inhibitors and Dyes

The DNA stain Hoechst 33342 (Molecular Probes) was added at a final concentration of 1 μg/ml after the fertilization envelopes were removed. Inhibitors were added at appropriate periods of the cell cycle and incubated 5 min prior to observation. Nocodazole was used at a final concentration of 20 μM from a 100× stock solution made fresh in DMSO. Latrunculin B (Sigma) was used at a final concentration of 20 μM from a 100× stock in DMSO. MG132 (Sigma) was added to the eggs 30 min before fertilization at a final concentration of 50 μM from a 100× stock in DMSO.

Fixation and Staining Procedure

Fixation and staining procedures were adapted from Foe and von Dassow (2008) and Strickland et al. (2004). Briefly, cells were fixed in 100 mM HEPES (pH 6.9), 50 mM EGTA, 10 mM MgSO4, 2% formaldehyde, 0.2% glutaraldehyde, 0.2% acrolein, 0.2% Triton X-100, and 400 mM dextrose for 45 min. Cells were then rinsed three times in PBT, treated with 0.1% NaBH4 in PBS to limit autofluorescence for 30 min, and finally blocked in 5% goat serum for 1 hr. Microtubule staining was performed using a primary anti-α-tubulin antibody, clone DM 1A (Sigma) at 1/8000 incubated overnight and a CY3-conjugated anti-mouse secondary antibody at 1/750 (Sigma) incubated during 5 hr. Actin staining was performed using Alexa fluor phalloidin (Molecular Probes) incubated during 1 hr.

Microscopy and Image Analysis

Microscopy was performed with an inverted wide-field fluorescence microscope with a motorized stage (Ludl Instrument). The objectives used were either a 10× 0.25 NA or a 40× 0.75 NA. Confocal imaging of microtubules was performed with a laser-scanning confocal microscope (LSM 710, Zeiss) with a 40× 1.3 NA oil objective. Images were acquired, processed, and analyzed with OpenLab (Improvision), Micro-manager, Image J, Zen (Zeiss), and MatLab (Mathwork).

Computational Modeling

All computational simulations were performed using MatLab (Mathwork).

SUPPLEMENTAL INFORMATION

Supplemental Information includes Extended Experimental Procedures, six figures, one table, and one movie and can be found with this article online at doi:10.1016/j.cell.2011.01.016.
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REFERENCES


Supplemental Information

EXTENDED EXPERIMENTAL PROCEDURES

Supplemental Model

(1) Estimation of Forces Stretching the Zygote Nucleus

We represent the nucleus as a spherical thin elastic shell with a surface modulus $K$ and a radius $r_N$. The force necessary to deform such a sphere into a prolate spheroid of long axis $r_N + \xi$ is given by:

$$F = \frac{K}{1 + \nu} \ln(r_N/r_F),$$

(S1)

where $\nu$ is the Poisson ratio of the elastic material and $r_F$ is the radius of the zone where the force is applied (there is no finite solution if the application zone is punctual). The conservation of the nuclear volume imposes:

$$\xi = r_N \left( \frac{r_N^2}{r_F^2} - 1 \right),$$

(S2)

with $r_N$ the aspect ratio of the deformed nucleus. Injecting Equation S2 into Equation S1 yields Equation 1 presented in the main text.

The nuclear lamina, which constitutes most of the mechanical properties of the nuclear surface, is usually considered as an incompressible elastic material and its Poisson ratio may be set to 0.5 (Vaziri and Mofrad, 2007). The radius of force application $r_F$, which corresponds to the zone of microtubules attachment on the nucleus, can be assumed to be around 1/10th of the nuclear radius (see Figure 2 and Figure S3). We use a surface modulus of 25 pN/\mu m as measured in *Xenopus* (Dahl et al., 2004) (this value may not vary by more than one order of magnitude in different cell types (Dahl et al., 2008)). Injecting the measured average aspect ratio of the deformed nucleus in normal round cells and a nuclear radius of 8.5 \mu m as measured in nocodazole treated cells into Equation 1, yields an estimate of the average total pulling force exerted by microtubules of:

$$F_0 \approx 10 \text{ pN}.$$

(2) Nuclear Force Scaling in Different Shapes

Computer Simulations. To elucidate how MT pulling forces may scale with MT length, we introduce a general scaling law of the form:

$$f(L) = k(\beta)L^\alpha.$$

(S3)

This approach aims at identifying a global behavior of the force generating system. Other general laws could also be considered (exponentials, logarithmic), but this power scaling law is the most naive that spans a large range of behavior using only one adjustable parameter (Hays et al., 1982).

Injecting Equation S3 into Equation 3 and setting $\alpha = 0$, which corresponds to stable orientations in rectangles and ellipse, yields the general equation for the total force exerted on the nucleus:

$$F_{max}(\beta, a/b) = \int \frac{1}{2} k(\beta)L^2 \cos(\theta) d\theta$$

(S4)

with $a/b$ the aspect ratio of the cell (Figure 3). Normalizing $F_{max}(\beta, a/b)$ by the force computed in a round cell, $F_0(\beta) = F_{max}(\beta, a/b = 1)$ allows eliminating the spring constant $k(\beta)$ and the MT density from the equation:

$$\bar{F}_{max}(\beta, a/b) = A \int \frac{1}{2} \left( \frac{L(\theta)}{R_0} \right) \cos(\theta) d\theta,$$

(S5)

with $A$, a constant, and $R_0$ the radius of the round cell. Equation S5 can be used to compute $\bar{F}_{max}$ for all values of $e$ and for any aspect ratio corresponding to the experiments. For each $e$, we extract how $\bar{F}_{max}$ scales with $a/b$ by fitting linearly the logarithmic plots to compute the slope $\alpha(e)$, as illustrated in Figure 5:

$$Ln\left( \bar{F}_{max}(\beta, a/b) \right) = \alpha(e) Ln\left( \frac{a}{b} \right) + \text{Cste}.$$

(S6)

This function $\alpha(e)$ that can be computed systematically in the model, is found to be monotonically increasing for the range of shapes that we study and thus can be inverted to extract $e$ from fitting the experimental data.

We extract experimental scaling from data on nuclear elongation (Figure 3C). We use Equation 1 to compute the normalized forces exerted on the nucleus in cells with varying aspect ratio, and fit a logarithmic plot to obtain experimental slopes $\alpha(e_{exp})$ (Figure 5). It is
important to note that, although the absolute force associated with nuclear deformation depends on several parameters, such as the actual nuclear surface modulus (Equation 1), the experimental normalized force, $\bar{F}$, only depends on the term $(\rho_N)^{2/3} - 1$:

$$\bar{F}_{\text{max}}(\beta_{\text{exp}} \frac{a}{b}) = \frac{(\rho_N(a/b))^{2/3} - 1}{(\rho_N(1)^{2/3} - 1)} \quad (S7)$$

Thus the main incertitude arising from using Equation 1 to extract experimental scaling, as presented in Figure 5, comes from assuming that the nucleus can indeed be considered as an elastic shell being pulled on locally at its surface (Rowat et al., 2005).

Logarithmic fits of the experimental plots yield values for $\sigma(\beta_{\text{exp}})$, which finally allow us to retrieve estimates of $\beta_{\text{exp}}$ for the rectangle and the ellipsoid cases reported in the main text.

**Analytical Solutions for the Case of the Ellipses.** For the simple case of the ellipsoidal shape, the total force and its dependence on cell aspect ratio and $\beta$ can be computed analytically with good approximation. This is valuable to understand which parameters are important, and also to compute the effect on integrating the MT forces in 3D.

To that aim, we keep the same geometrical setting as in Figure 3 and Figure 5 and use a total angular width for the aster $F = 180^\circ$.

The angle $\alpha$ is set to 0 and the angle $\theta$ represents the angular coordinate from the long axis of the ellipse. The total force exerted by the MTs on the nucleus is:

$$F_{\text{max}}(\beta, \frac{a}{b}) = \frac{1}{2} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} k(\beta)(L(\theta))^{\frac{3}{2}} \cos(\theta) d\theta.$$  \hspace{1cm} (S8)

We neglect the volume of the nucleus so that MTs now extend from the cell’s center of mass to the cortex. We can start to solve the case of the round cell, for which $L(\theta) = R_0$:

$$F_{\text{max}}(\beta, 1) = \frac{1}{2} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (L(\theta))^{\frac{3}{2}} \cos(\theta) d\theta.$$  \hspace{1cm} (S9)

yielding:

$$F_{\text{max}}(\beta, 1) = 2 \rho k(\beta) R_0^2.$$  \hspace{1cm} (S10)

For ellipses of aspect ratio $a/b$, the total force $F_{\text{max}}(\beta, a/b)$ can be renormalized using Equation S10 to eliminate $k$ and $\rho$:

$$\bar{F}_{\text{max}}(\beta, \frac{a}{b}) = \frac{1}{2} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\frac{L(\theta)}{R_0}\right)^{\frac{3}{2}} \cos(\theta) d\theta.$$  \hspace{1cm} (S11)

Geometrical considerations in the ellipse allow to derive a simple expression for $L(\theta)$:

$$L(\theta) = \frac{b}{\sqrt{1 - \left(1 - \left(\frac{b}{a}\right)^2\right) \cos^2(\theta)}}.$$  \hspace{1cm} (S12)

The conservation of the surface area between ellipsoid shapes of different aspect ratio further imposes:

$$\frac{b}{R_0} = \frac{b}{a}.$$  \hspace{1cm} (S13)

After replacing Equations S12 and S13 into Equation S11 and algebraic simplifications, we obtain the general expression for the normalized force:

$$\bar{F}_{\text{max}}(\beta, \frac{a}{b}) = \frac{1}{2} (\frac{a}{b})^{\frac{3}{2}} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos(\theta) d\theta}{\left(1 - \left(1 - \left(\frac{b}{a}\right)^2\right) \cos^2(\theta)\right)^{\frac{3}{2}}}.$$  \hspace{1cm} (S14)

Exact solutions of Equation S14 can be computed analytically for $\beta = 1$ and $\beta = 2$:
Fitting Equation S15 and Equation S16 yields the values: $\sigma(1) = 0.04$ and $\sigma(2) = 0.22$. Further, general solutions for all $\beta$ value can be computed to obtain the complete evolution of $\sigma(\beta)$. Comparisons with the computer-based method presented before yield excellent agreement (better than 5%) for $\sigma(\beta)$.

Analytical Solutions for the Case of the Ellipses Considering the 3D Distribution of MTs. Because nuclear deformation results from forces exerted by all MTs in the 3D volume of the cell, we aimed at testing the influence of using a 3D model on the estimation of $h/2$ (h is the chamber’s height), semi-short axis b and semi-long axis a. Geometrical considerations allow to derive $L(\theta, \phi)$ for any ellipsoid, where $\phi$ is the inclination angle and $\theta$ the azimuth in spherical coordinates:

$$L(\theta, \phi) = \frac{h/2}{\sqrt{1 + \left(\frac{a/b}{4R_0^2} h^2 \left(1 - \left(\frac{b}{a}\right)^2 \cos^2(\theta)\right) - 1\right) \sin^2(\phi)}}$$  \hspace{1cm} (S17)

The force exerted by each MT and projected along the symmetry axis ($\phi = \pi/2$, $\theta = 0$), is now obtained by:

$$f(L) = k(\beta) L^2 \cos(\theta) \sin(\phi).$$  \hspace{1cm} (S18)

Summing over the 3D volume of the half-ellipsoid yields the total 3D force:

$$F_{\text{max}}(\beta, b, b) = \int_{\theta = 0}^{\pi/2} \int_{\phi = 0}^{\pi/2} f(L) \rho d\Omega.$$  \hspace{1cm} (S19)

With $d\Omega$ the unit solid angle ($d\Omega = \sin(\phi)d\phi d\theta$). Combining Equations S17–S19 yields the general expression for the total force as a function of $\beta$ and the aspect ratio $a/b$:

$$F_{\text{max}}(\beta, a/b) = k(\beta) \int_{\phi = 0}^{\pi/2} \sqrt{\frac{\sin^2(\phi) \cos(\theta)}{1 + \left(\frac{a/b}{4R_0^2} h^2 \left(1 - \left(\frac{b}{a}\right)^2 \cos^2(\theta)\right) - 1\right) \sin^2(\phi)}} d\theta d\phi$$  \hspace{1cm} (S20)

Normalizing and computing Equation S20, allows to compute theoretically $\sigma(\beta)$ which, as before, yields an estimate of experimental exponent $\beta_{\text{exp}}$. For the ellipsoid, this calculation provides an estimate in the 3D context of $\beta_{\text{ell}} = 4.4 \pm 1.1$. Thus this analytical calculation suggest that the values reported in the main text may be underestimating the true scaling by around 20%–40%.

(3) Metaphase Spindle Rotation

The dynamic equation of rotation of the nucleus or the metaphase spindle under a given torque can be written as follows:

$$k \frac{d\alpha(t)}{dt} = T(\alpha).$$  \hspace{1cm} (S21)

Where k is a rotational friction coefficient that we approximate using a spherical description for the nucleus or the metaphase spindle: $k = 8\pi\eta r_N^2$, with $r_N$ the nucleus/spindle radius and $\eta$ the viscosity of the cytoplasm. Using our normalized notation we can rewrite Equation S21 as:

$$\tilde{k} \frac{d\alpha(t)}{dt} = \tilde{T}(\alpha)$$  \hspace{1cm} (S22)

with $\tilde{k} = 8\pi\eta r_N^2/F_0$ that we estimate to be around 2 s by assuming a viscosity of 0.01 Pa.s (Valentine et al., 2005) $r_N = 8.5 \mu m$ and $F_0 = 10$ pN.
From Equation S22, we can compute the theoretical time \( t_{th} \) it takes to rotate from an initial position \( \alpha_0 \) to \( \alpha(t_{exp}) \) as:

\[
t_{th} = e^{\int_{\alpha_0}^{\alpha(t_{exp})} \frac{d\alpha}{\bar{T}(\alpha)}}.
\]  
(S23)

As we know the distribution of torques, \( \bar{T}(\alpha) \), for any given shape, the time \( t_{th} \) can be computed by inferring the experimental evolution of \( \alpha(t_{exp}) \). This allows us to plot and compare the experimental \( \alpha(t_{exp}) \) and theoretical \( \alpha(t_{th}) \) evolution of the spindle angle reorientation to the force axis in all our cells, as presented in Figure 7B and Figure S6. For each point we then compute the difference in time \( t_{exp} - t_{th} \), average it over all conditions, and study how this average will change as a function of the number of microtubules (Figure 7G). At a fixed force per MT length, the global torque will increase with the number of MTs, and thus the spindle will rotate faster.

**4) Influence of Different Parameters on the Output of the Computational Model**

Our computational model, as presented in the main text and Figure 4, Figure 5, Figure 6, and Figure 7, uses a set of 7 different parameters that are either extracted from experiments, silent in the model (they do not affect the output), or fitted from experimental results. Table S1 associated with Figure S4 and Figure S5, provides information on source, values, units, and impact of these diverse parameters.

**SUPPLEMENTAL REFERENCES**


Figure S1. Chamber Design and Division Pattern in Different Geometry, Related to Figure 1

(A) Micrograph of the PDMS chambers used to shape urchin eggs. Scale bar, 1 mm.
(B) Sea Urchin embryo developing into PDMS chambers. 1 cell and 16 cells stages are shown. Scale bar, 50 μm.
(C) Summary of all observed division-plane position and orientation in different shapes. The number of observed cells in each condition varies from 15 to 30 and the bin size is adapted to this number in each rose plot.
Figure S2. Conservation of the Division Axis from Interphase to Cytokinesis, Related to Figure 1

(A) Different time-lapse sequences of embryos going through interphase, metaphase, anaphase, and cytokinesis in different geometries. The typical elapsed time from the first to the last image is 30–40 min. Note that the division axis is set at interphase and that the furrow cleaves the cell perpendicular to this axis.

(B) Time-lapse illustrating that the cleavage furrow does not reorient or slide during cytokinesis in different geometries. Scale bars, 50 μm.
Figure S3. In Situ Immunofluorescence of Actin and Microtubules in Eggs with Different Shapes, Related to Figure 2

(A) Schematic representing the microfluidic-based method used to perfuse cells without affecting the shape of the eggs in the wells. The pressure imposed by adding perfusion liquids in the large holes will generate a flow of liquid around the eggs, but may also cause the PDMS to slightly lift from the coverslip. This lifting can cause the egg to flow out of the micro-well and lose its imposed shape. Thus, this pressure drop needs to be adjusted carefully and slowly, while monitoring cells in chambers.

(B) Time-lapse sequence of an egg fixed, permeabilized, and stained for actin inside the chambers. Note that the shape of the cell remains almost constant.

(C) Confocal images of cells in chambers fixed and stained in situ for actin in the presence of 1% DMSO or 20 μM Latrunculin B. Images are stacks of 20 mid-section slices that cover a total depth of 20 μm. Exposure and contrast is the same in these two images.

(D) Confocal images of cells outside chambers fixed and stained for tubulin in the presence of 1% DMSO or 20 μM Latrunculin B.

(E) Confocal images of cells in chambers fixed and stained in situ for tubulin (green) and DNA (blue), in the presence of 1% DMSO. Images are stacks of 20 mid-section slices that cover a total depth of 10 μm.

(F) The angular distribution of MTs emanating from the centrosomes is roughly constant. Plots are obtained from four different cells. Scale bars, 20 μm.
Figure S4. Influence of MT Force-Length Scaling on Axis Orientation, Related to Figure 4
(Top) Schematic representation of different proposed models for MT-pulling force generation, and corresponding scaling of the force, f, for each MT as a function of MT length, L.
(Bottom) Model prediction of torque and orientation density probability of the division axis in the depicted shapes, for different force-generation models (plot lines colors correspond to box colors in the schematics). The experimental frequency histograms of division axis are represented at the bottom.
Figure S5. Testing the Influence of Different Model Parameters, Related to Figure 4

(A) Influence of the parameter C on shape-sensing sensitivity. (Top) DIC images of divided eggs in different geometries. (Bottom) Plot of the experimental and theoretical division-plane orientation to the y axis, \( \sin(a_{\text{div}}) \cos(a_{\text{div}}) \), as a function of the geometrical aspect ratio. The orientation to the y axis is 1 if all cells cleave parallel to the y axis and 0 for a random distribution of cleavage planes. Each experimental point is averaged on at least 15 different cells in a given shape. Theoretical average and standard deviation are computed as:

\[
\left( \frac{\sin(a_{\text{div}})}{C_0} \cos(a_{\text{div}}) \right)
\]

respectively. Note that high values of C make the system insensitive to shape changes while low values of C make the system too sensitive.

(B) Influence of the number of MTs in the aster, N. Note that for N > 10 the distributions are almost identical.

(C) Influence of the aster angle on division-axis positioning in the square and rectangle geometries. Note that at large F (close to \( 360^\circ \)) the square’s preferred division plane becomes the diagonal. The experimental histograms of division axis are represented at the bottom.

(D) Influence of the homogeneity in MT nucleation angular density, \( \rho \). The general law for the non-homogeneous angular density is taken as a hat-function parameterized by \( \lambda: \rho(\theta) = -\text{sign}(\theta) + 1 - 45/\theta - 90\text{sign}(\theta) + \lambda \) and combination of values for \( \lambda = 0-1 \), are probed. Note that at very small \( \lambda \), differences in the distribution for the square begin to appear.

(E) Influence of the nuclear size, \( r_N \), on division-axis positioning in the square and rectangle geometries. Note that at large sizes, when the nuclear radius is close to half of the cell radius, the square’s preferred division plane becomes the diagonal.
Figure S6. Experimental and Theoretical Metaphase Spindle Reorientation to the Force Axis, Related to Figure 7
Experimental and theoretical plots of the reorientation of the spindle axis angle to the theoretical force axis. The theoretical plots are computed by assuming a ratio of metaphase to interphase MT number of 1/150 (see Figure 7G).